THE GROWTH AND DECAY OF SHIP WAVES

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PMM Vol.27, No.4, 1963, pp. 725-730

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(Received January 7, 1963)

The spatial problem of the growth and decay of surface waves, which arise as a result of the rectilinear motion of a pressure system with constant velocity c along a free surface, is investigated. Analogous problems are examined in the case of steady motions in [1-4].

1. Starting with the moment of time $t = 0$, let pressure of the form $p = p_0 a(x, y)$ be applied to the horizontal free surface of a fluid occupying the half-plane $z < 0$ and flowing in the positive direction of the x-axis with the velocity c. Assuming the motion of the fluid to be irrotational, we write the velocity potential in the form

$$\Phi(x, y, z, t) = cx + \varphi(x, y, z, t)$$

The potential of the perturbed motion $\varphi(x, y, z, t)$ must satisfy the following conditions [2]:

$$\Delta\varphi = 0, \qquad z < 0 \tag{1.1}$$

$$\varphi_{tt} + g\varphi_z + 2c\varphi_{xt} + c^2\varphi_{xx} = -c\rho^{-1}p_0 a_x, \qquad z = 0 \tag{1.2}$$

$$\varphi(x, y, z, 0) = 0, \qquad \varphi_t + c\varphi_x = -p_0\rho^{-1}a, \qquad z = 0, \qquad t = 0 \tag{1.3}$$

In addition, the elevation $\zeta(x, y, t)$ of the free surface of the fluid is given by the formula

$$\zeta = -g^{-1}[\varphi_t + c\varphi_x + p_0\rho^{-1}a]_{z=0} \tag{1.4}$$

Conditions (1.3) express the fact that at the initial moment of time there is no perturbed motion and the free surface is horizontal. Applying Fourier transforms with respect to the variables x and y to (1.1) to (1.4), we obtain

$$\Phi_{zz} - (m^2 + n^2)\,\Phi = 0, \qquad z < 0 \tag{1.5}$$

$$\Phi_{tt} + g\Phi_z + 2cim\Phi_t - m^2c^2\Phi = -c\rho^{-1}p_0\, imA, \qquad z = 0 \tag{1.6}$$

$$\Phi\,(m, n, z, 0) = 0, \qquad \Phi_t + imc\Phi = -p_0\rho^{-1}A, \qquad z = 0, \qquad t = 0 \tag{1.7}$$

$$Z = -g^{-1}\,(\Phi_t + cim\Phi + p_0\rho^{-1}A)_{z=0} \tag{1.8}$$

where Φ, A, Z are the Fourier transforms of the functions φ, a, ζ. The solution of equation (1.5) which satisfies the condition $\Phi \to 0$ as $z \to -\infty$ has the form

$$\Phi = B\,(m, n, t)\, e^{z\sqrt{m^2+n^2}} \tag{1.9}$$

Satisfying conditions (1.6) to (1.7), we obtain

$$B_{tt} + 2cimB_t + (g\sqrt{m^2 + n^2} - m^2c^2)\,B = -cim\rho^{-1}p_0A$$
$$B\,(m, n, 0) = 0, \qquad B_t\,(m, n, 0) = -p_0\rho^{-1}A$$

Hence

$$B\,(m, n, t) = B_0 + B_1e^{ik_1t} + B_2e^{ik_2t} \tag{1.10}$$

$$B_0 = \frac{cim_0p_0A}{\rho\,[c^2 - \alpha^2]}, \qquad B_{1,2} = -\frac{ip_0A}{2\rho\,[cm \mp \alpha]}, \quad k_{1,2} = -cm \pm \alpha, \quad \alpha = \sqrt[4]{g^2\,(m^2 + n^2)}$$

Applying inverse Fourier transforms, we find from formulas (1.8) and (1.9)

$$\zeta = \frac{p_0}{4\pi\rho g}\lim_{z\to -0}\left[\iint\limits_{-\infty}^{\infty} f\,(m, n, t)\, e^{i\,(mx+ny)+z\sqrt{m^2+n^2}}\,dm\,dn\right] \tag{1.11}$$

$$f\,(m, n, t) = A\,(m, n)\left[\frac{2c^2m^2}{c^2m^2 - \alpha^2} - \frac{\alpha}{cm - \alpha}\,e^{ik_1t} + \frac{\alpha}{cm + \alpha}\,e^{ik_2t}\right]$$

$$\varphi = \frac{1}{2\pi}\iint\limits_{-\infty}^{\infty} B\,(m, n, t)\, e^{z\sqrt{m^2+n^2}}\,dm\,dn \tag{1.12}$$

2. Formulas (1.11) and (1.12) give an exact solution to the posed problem for an arbitrary function $a(x, y)$. We shall carry out an investigation of these formulas for a function $a(x, y)$ which is equal to unity in the square $|x| < b$, $|y| < b$ and is equal to zero outside of this square. In this case

$$A\,(m, n) = \frac{2\sin mb \sin nb}{\pi mn} \tag{2.1}$$

We shall assume further that b is small but that p_0 is so large that the total pressure force $P = p_0 b^2$ has a finite value. Taking (2.1) into consideration we write expressions (1.11) and (1.12) in the following form:

$$\varphi = -\frac{p_0}{2\pi\rho}\,\mathrm{Im}\left[\int\limits_{-\infty}^{\infty}\int \frac{A(m,n)}{(cm-\alpha)}\left(1-e^{ik_1t}\right)e^{i(mx+ny)+z\sqrt{m^2+n^2}}\,dm\,dn\right] \tag{2.2}$$

$$\zeta = \frac{p_0}{2\pi\rho g}\lim_{z\to -0}\left[\mathrm{Re}\int\limits_{-\infty}^{\infty}\int \frac{A(m,n)}{cm-\alpha}\left(cm-\alpha e^{ik_1t}\right)e^{i(mx+ny)+z\sqrt{m^2+n^2}}\,dm\,dn\right] \tag{2.3}$$

We shall introduce the notations

$$m = \varkappa r\cos\theta, \qquad n = \varkappa r\sin\theta, \qquad x = R\cos\psi, \qquad y = R\sin\psi$$
$$\varkappa = gc^{-2}, \quad t_1 = \tan c^{-1}, \quad z_1 = \varkappa z, \quad b_1 = \varkappa b, \quad R_1 = \varkappa R$$

Expression (2.3) takes the form

$$\zeta = \frac{p_0}{\pi^2\rho g}\lim_{z\to -0}\mathrm{Re}\left[\int\limits_0^{\infty} I(r)\,e^{zr}\,dr\right], \qquad I(r) = \int\limits_{-1/2\pi}^{3/2\pi}\xi(\theta)\,e^{irR\cos(\theta-\psi)}\,d\theta \tag{2.4}$$

where

$$\xi(\theta) = f_1(\theta)\left[\sqrt{r}\cos\theta - e^{-i\sqrt{r}\,(\sqrt{r}\cos\theta-1)t}\right]$$
$$f_1(\theta) = \frac{\sin(br\cos\theta)\sin(br\sin\theta)}{r\left(\sqrt{r}\cos\theta-1\right)\sin 2\theta} \tag{2.5}$$

Here the index of unity on R, z and b has been omitted for simplicity in writing. Since the function $\xi(\theta)$ does not have singularities along the path of integration $-1/2\,\pi \leqslant \theta \leqslant 3/2\,\pi$ for any r, including also $r = 0$ since $\xi(\theta) \to 0$ as $r \to 0$, the original path of integration in formula (2.5) can then be replaced by the path L which runs along the real axis with a bypass around the point θ_1 by means of a small semicircle in the lower half-plane and around the point θ_2 in the upper half-plane, where

$$-{}^1/_2\pi < \theta_1 < 0, \qquad 0 < \theta_2 < {}^1/_2\pi, \qquad \cos\theta_1 = \cos\theta_2 = r^{-1/2}$$

Along the chosen contour L we have $\mathrm{Re}\,[-ir\cos\theta] \leqslant 0$. The integral (2.5) can now be written as

$$I = I_1 - I_2 \tag{2.6}$$

$$I_1 = \int\limits_{(L)} f_1(\theta)\sqrt{r}\,e^{irR\cos(\theta-\psi)}\cos\theta\,d\theta, \qquad I_2 = \int\limits_{(L)} f_1(\theta)\,e^{iRrM(\theta)}\,d\theta \tag{2.7}$$

$$M(\theta) = \cos(\theta-\psi) - \nu\left[\cos\theta - r^{-1/2}\right], \qquad \nu = tR^{-1} \tag{2.8}$$

In an analogous manner expression (2.2) can be written in the following form:

$$\varphi = -\frac{cp_0}{\pi^2\rho g}\,\mathrm{Im}\left[\int_0^\infty (K_1(r) - K_2(r))\, e^{zr}\frac{1}{\sqrt{r}}\,dr\right] \tag{2.9}$$

$$K_1 = \int_{(L)} f_1(\theta)\, e^{irR\cos(\theta-\psi)}\,d\theta, \qquad K_2 = \int_{(L)} f_1(\theta)\, e^{iRrM(\theta)}\,d\theta \tag{2.10}$$

Evaluating the integrals (2.7) for large values of R by the method of stationary phase [1,2] for $r > 1$, we find

$$I_1 = a_1 + a_2 + a_3, \qquad I_2 = b_1 + b_2 + b_3 \tag{2.11}$$

$$a_1 = -2\pi i\,[\mathrm{res}\,\psi_1(\theta)]_{\theta_2}, \quad \cos\psi > 0, \quad \sqrt{r} > \sec\psi$$

$$a_2 = 2\pi i\,[\mathrm{res}\,\psi_1(\theta)]_{\theta_1}, \quad \cos\psi > 0, \quad r > 1; \quad \cos\psi < 0, \quad \sqrt{r} < -\sec\psi$$

$$a_3 = R^{-1/2} d_1(r)\, e^{iRq_1(r)}$$

$$b_1 = 2\pi i\,[\mathrm{res}\,\psi_2(\theta)]_{\theta_1}, \quad 0 < \psi < \pi, \quad \nu < \nu_1, \quad r > 1$$

$$b_2 = -2\pi i\,\mathrm{res}\,[\psi_2(\theta)]_{\theta_2}, \quad \cos\psi > \nu > \cos\psi - (r-1)^{-1/2}\sin\psi, \quad r > 1$$

$$b_3 = R^{-1/2} d_2(r)\, e^{iRq_2(r)}$$

$$\nu_1 = (\sqrt{r-1}\cos\psi + \sin\psi)\,/\,\sqrt{r-1}$$

Here the integrands of the integrals (2.7) are denoted by $\Psi_1(\theta)$ and $\Psi_2(\theta)$; $d_1(r)$, $d_2(r)$ denote functions which do not have singularities; and the functions $q_1(r)$, $q_2(r)$ are real for $r > 0$.

From formulas (2.11), (2.6) and (2.4) we find

$$\zeta = \frac{p_0}{\pi^2\rho g}\lim_{z\to 0}\mathrm{Re}\left[\sum_{k=3}^{11} I_k\right] \tag{2.12}$$

Here the I_k are determined by the formulas

$$I_3 = \pi i\int_1^\infty \xi_1(r)\, e^{[iR\sqrt{r}(\cos\psi - \sqrt{r-1}\sin\psi)]}\,dr \quad \begin{pmatrix}\cos\psi > 0\\ \nu > \nu_1\end{pmatrix} \tag{2.13}$$

$$I_4 = \int_0^1\int_{-1/2\pi}^{3/2\pi} f_1(\theta)\sqrt{r}\cos\theta\, e^{irR\cos(\theta-\psi)+zr}\,d\theta\,dr, \quad I_5 = \int_0^1\int_{-1/2\pi}^{3/2\pi} f_1(\theta)\, e^{iRrM(\theta)+zr}\,d\theta\,dr \tag{2.14}$$

$$I_6 = \pi i\int_1^\Psi \xi_1(r)\, e^{iR\sqrt{r}(\cos\psi - \sqrt{r-1}\sin\psi)}\,dr \qquad \begin{pmatrix}\cos\psi < 0\\ \nu > \nu_1\end{pmatrix} \quad \left(\Psi = \frac{1}{\cos^2\psi}\right)$$

$$I_7 = -\pi i\int_\Psi^\infty \xi_1(r)\, e^{iR\sqrt{r}(\cos\psi - \sqrt{r-1}\sin\psi)}\,dr \qquad \begin{pmatrix}\cos\psi < 0\\ \nu < \nu_1\end{pmatrix}$$

(2.15)

$$I_8 = \quad \pi i \int_{\Psi}^{\infty} \xi_1(r)\, e^{iR\sqrt{r}\,(\cos\psi + \sqrt{r-1}\sin\psi)}\, dr \qquad \begin{pmatrix} \cos\psi > 0 \\ \nu > 0 \end{pmatrix}$$

$$I_9 = -\pi i \int_{1}^{\infty} \xi_1(r)\, e^{iR\sqrt{r}\,(\cos\psi + \sqrt{r-1}\sin\psi)}\, dr \qquad \left(\cos\psi < \nu < \cos\psi - \frac{\sin\psi}{\sqrt{r-1}}\right)$$

$$I_{10,11} = \frac{1}{\sqrt{R}} \int_{1}^{\infty} d_{1,2}(r)\, e^{iRq_{1,2}(r)+zr}\, dr,$$

In formulas (2.13) to (2.15)

$$\xi_1(r) = \frac{\sin b\sqrt{r}\,\sin b\sqrt{r(r-1)}}{r-1}\, e^{zr}$$

Carrying out evaluations of the integrals I_k ($k = 4, \ldots, 11$) for large values of R, and taking into consideration that $d_1(r)$, $d_2(r)$ and the integrands in I_4 and I_5 according to (2.14) do not have singularities in the region of integration [5], and that the I_k ($k = 6,7,8,9$) do not have stationary points on the path of integration, we find that each of these integrals is of order not lower than R^{-1} and that therefore

$$I_4 + I_5 + \ldots + I_{11} = 0\,(R^{-1})$$

We shall pass on to the investigation of the integral I_3; we represent it in the following form:

$$I_3 = 2\pi i \int_{0}^{\infty} \frac{\cosh r \sin(b\cosh r)\sin(b\cosh r\sinh r)}{\sinh r}\, e^{iRN(r)+z\cosh^2 r}\, dr \qquad (2.16)$$

Here

$$N(r) = \cosh r\,(\cos\psi - \sinh r\sin\psi), \qquad \cos\psi > 0, \qquad \nu > \nu_1 = \frac{\sinh r\cos\psi + \sin\psi}{\sinh r}$$

The equation $N'(r) = 0$ has the roots

$$\sinh r_{1,2} = \frac{\cos\psi \pm \sqrt{9\cos^2\psi - 8}}{4\sin\psi}$$

Since the stationary points of the integrand of (2.16) occur only for $0 < \Psi < 19°28'$, we obtain for I_3 the asymptotic expression for large values of R

$$I_3 = \begin{cases} 2\pi i R^{-1/2}\,\Sigma A_k \exp\{iR\,[N(r_k) + (-1)^{k_1}/_4\pi] + z\cosh^2 r_k\} & (0 < \psi < 19°28') \\ 2\pi i R^{-1/3} A_3 \exp\,[iRN(r_3) + z\cosh^2 r_3] & (\psi = 19°28') \\ O\,(R^{-1}) & (19°28' < \psi < \pi) \end{cases} \qquad (2.17)$$

$$A_k = \left[\frac{\sqrt{\cosh r}\,\sqrt{2\pi}\sin(b\cosh r)\sin(b\cosh r\sinh r)}{\sinh r\,\sqrt[4]{9\cos^2\psi - 8}}\right]_{r_k} \qquad (k = 1, 2)$$

$$A_3 = 3^{5/12}2^{1/4}\Gamma(1/_3)\sin(1/_2\sqrt{3}\,b)\sin(1/_2\sqrt{2}\,b), \qquad \sinh r_3 = 1/_2\sqrt{2}$$

From formulas (2.12), (2.15) and (2.17) we find the expressions for ζ for large κR

$$\zeta = \begin{cases}\eta_1 + \eta_2 & (0 < \psi < 19°28')\\ O\,[(\kappa R)^{-1}] & (19°28' < \psi \leqslant \pi)\end{cases} \qquad (2.18)$$

$$\eta_{1,2} = \begin{cases}B_{1,2}(\kappa R)^{-1/2}\sin[N(r_{1,2})\kappa R \mp 1/_4\pi] & (R < ctu_{1,2}(\psi))\\ O\,[(\kappa R)^{-1}] & (R > ctu_{1,2}(\psi))\end{cases}$$

For $\Psi = 19°28'$ we have

$$\zeta = \begin{cases}B_3(\kappa R)^{-1/3}\sin(1/_2\sqrt{3}\,\kappa R) & (R < 1/_2\sqrt{2}\,ct)\\ O\,[(\kappa R)^{-2/3}] & (R > 1/_2\sqrt{2}\,ct)\end{cases} \qquad (2.19)$$

$$B_k = -\frac{2p_0}{\pi\rho g}A_k \quad (k = 1, 2, 3), \qquad u_{1,2}(\psi) = \frac{\cos\psi \pm \sqrt{9\cos^2\psi - 8}}{4\sin^2\psi + (\cos\psi \pm \sqrt{9\cos^2\psi - 8})\cos\psi}$$

We set $\Psi = 0$ in the integrals (2.7) and, finding their asymptotic value for large κR, we have the following expression for ζ for $\Psi = 0$:

$$\zeta = \begin{cases}B_4(\kappa R)^{-1/2}\sin(\kappa R + 1/_4\pi) & (R < ct)\\ O\,[(\kappa R)^{-1}] & (R > ct)\end{cases} \quad \left(B_4 = -\frac{2\sqrt{2}\,p_0 b\sin b}{\sqrt{\pi}\,\rho g}\right) \qquad (2.20)$$

For a pressure which is concentrated at the origin of the coordinate system, i.e. in the case $b \to 0$, $p_0 \to \infty$, $b^2 p_0 = P = \mathrm{const}$, formulas (2.18) to (2.20) remain valid themselves but the expressions for B_k in this case take the following form:

$$B_{1,2} = -\left[\frac{2\sqrt{2}\,\kappa^2 P(\cosh r)^{5/2}}{\sqrt{\pi}\,\rho g\sqrt{9\cos^2\psi - 8}}\right]_{r_{1,2}}, \qquad (2.21)$$

$$B_3 = -\frac{2^{-1/4}3^{7/12}\kappa^2 P}{\pi\rho g}, \qquad B_4 = -\frac{2\sqrt{2}\,\kappa^2 P}{\sqrt{\pi}\,\rho g}$$

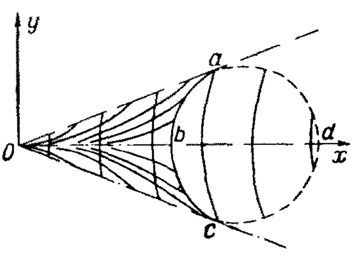

Fig. 1.

Thus, the principal perturbations of the free surface are concentrated inside the angle $19°28' \geqslant \Psi \geqslant -19°28'$ on the left of the curve adc (Fig. 1) whose equation has the form $R = u_1(\psi)ct$. The free surface with-the region $oabco$ will be covered by stationary ship waves which represent the sum total of the longitudinal waves η_2 and the transverse waves η_1. Moreover, the equation of the curve abc has the form $R = ctu_2(\psi)$.

The free surface within the region *abcda* will be covered only by the transverse waves η_1. The elevation of the fluid within the angle on the right of the curve *adc* is of the same order as the elevation outside of the angle. The values of $u_{1,2}(\psi)$ are given in the table.

ψ	u_1	u_2	ψ	u_1	u_2
0°	1.000	0.500	14°	0.900	0.556
3°	0.996	0.502	16°	0.862	0.580
6°	0.983	0.509	18°	0.807	0.620
9°	0.962	0.520	19°	0.763	0.655
12°	0.929	0.538	19°28	0.706	0.706

3. We shall now investigate the problem of the decay of the stationary waves generated by the moving pressures under consideration after stopping the action of the pressures. The expressions for the potential and the fluid elevation which we have obtained in the previous paragraph can be written as

$$\varphi = \varphi_1 - \varphi_2, \qquad \zeta = \zeta_1 - \zeta_2, \qquad \varphi_{1,2} = -\frac{cp_0}{\pi^2\rho g}\,\mathrm{Im}\int_0^\infty K_{1,2}(r)\,e^{zr}dr \tag{3.1}$$

$$\zeta_1 = \frac{p_0}{\pi^2\rho g}\lim_{z\to -0}\mathrm{Re}\int_0^\infty I_1(r)\,e^{zr}\,dr - \frac{p_0 a(x, y)}{\rho g}, \quad \zeta_2 = \frac{p_0}{\pi^2\rho g}\lim_{z\to -0}\mathrm{Re}\int_0^\infty I_2(r)\,e^{zr}\,dr \tag{3.2}$$

where $I_{1,2}$, $K_{1,2}$ are given by formulas (2.7) and (2.10). The contour L is chosen so that ζ_2 and φ_2 tend to zero as $t \to \infty$; therefore, the stationary motion will be determined by the potential φ_1, and the fluid elevation in this case will be determined by formula (3.2). Taking the moment of stopping the action of the pressures as the initial time ($t = 0$), we have the following equations to determine the function $\varphi(x, y, z, t)$, the potential of the decaying wave motion in a coordinate system moving with velocity c in the negative direction of the x-axis:

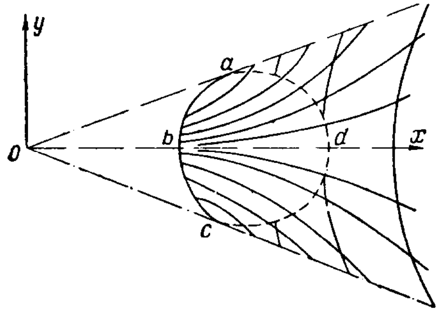

Fig. 2.

$$\Delta\varphi = 0 \qquad z < 0 \tag{3.3}$$

$$\varphi_{tt} + g\varphi_z + 2c\varphi_{xt} + c^2\varphi_{xx} = 0, \qquad z = 0$$

$$\varphi(x, y, z, 0) = \varphi_1(x, y, z, 0) \tag{3.4}$$

$$\varphi_t + c\varphi_x = \varphi_{1t} + c\varphi_{1x} + p_0\rho^{-1}a, \qquad t = 0, \qquad z = 0 \tag{3.5}$$

Since the function $\varphi_1 - \varphi_2$ satisfies the initial conditions (1.3), it is then obvious that

$$\varphi(x, y, z, t) = \varphi_2(x, y, z, t)$$

will satisfy all the conditions (3.3) to (3.5) therefore, the fluid elevation in the decaying motion will have the form $\zeta = \zeta_2$, where ζ_2 is given by formula (3.2). Using the results of the investigation of the integral I_2 given by formula (2.11), we find the following final expression for the fluid elevation in the decaying motion for large κR in a coordinate system moving with the velocity c

$$\zeta = \begin{cases} \eta_1 + \eta_2 & (0 < \psi < 19°28') \\ O(\varkappa^{-1}R^{-1}) & (19°28' < \psi \leqslant \pi) \end{cases}$$

$$\eta_{1,2} = \begin{cases} B_{1,2}(\varkappa R)^{-1/2} \sin [N(r_{1,2}) \mp {}^1/_4\pi] & (R > ctu_{1,2}(\psi)) \\ O(\varkappa^{-1}R^{-1}) & (R < ctu_{1,2}(\psi)) \end{cases}$$

$$\zeta = \begin{cases} B_3(\varkappa R)^{-1/3} \sin ({}^1/_2 \sqrt{3}\varkappa R) & (R > {}^1/_2\sqrt{2}\, ct, \quad \psi = 19°28') \\ O[(\varkappa R)^{-2/3}] & (R < {}^1/_2\sqrt{2}\, c, \quad \psi = 19°28') \end{cases}$$

$$\zeta = \begin{cases} B_4(\varkappa R)^{-1/2} \sin (\varkappa R + {}^1/_4\pi) & (R > ct, \quad \psi = 0) \\ O(\varkappa^{-1}R^{-1}) & (R < ct, \quad \psi = 0) \end{cases}$$

Here $N(r)$, B_k, r_k and u_k are given by formulas (2.16), (2.19) and (2.21).

The principal perturbations in the decaying motion will be contained inside of the angle $19°28' \geqslant \Psi \geqslant -19°28'$ to the right of the curve *abc* (Fig. 2). The region *abcd* will be covered only by longitudinal waves and the region to the right of the curve *adc* will be covered by the complete system of ship waves.

As $t \to \infty$ the amplitude of oscillations of the free surface in all regions of the fluid tends to zero like a quantity of order $(gc^{-1}t)^{-1}$.

BIBLIOGRAPHY

1. Sretenskii, L.N., *Teoriia volnovykh dvizhenii zhidkosti (Theory of the wave motions of a fluid)*. ONTI, 1936.

2. Stoker, J.J., *Volny na vode (Water waves)*. IL, 1959.

3. Hogner, E., A contribution to the theory of ship waves. *Arkiv för mathematik, astronomi och fysik*, 17 : 12, 1-50 (98), 1922.

4. Timman, R., The wave pattern of a moving ship. "Simon Stevin", 35, Nos. 1-2, 1961.

5. Fedoriuk, M.V., Metod statsionarnoi fazy dlia mnogomernykh integralov (Method of stationary phase for multi-dimensional integrals). *Zh. vychisl. matem. i matem. fiz.*, Vol. 2, No. 1, 1962.

Translated by **R.D.C.**